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SOLID-PHASE COMBUSTION IN A PLANE STRESS STATE.

2. STABILITY TO SMALL PERTURBATIONS

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Stability of a plane reaction front in the solid phase to small perturbations under conditions of a generalized plane stress state is studied with allowance of the coupled character of heat transfer and deformation processes and possible changes in the reaction rate under the action of internal stresses without external mechanical loading. The problem is solved analytically by the method of perturbations. Conditions of the loss of stability of various conversion regimes in some limiting cases are studied for different technological and physical parameters.

Key words: *plane reaction front, stability of conversion regimes, generalized plane stress state.*

Introduction. A model of self-sustained synthesis of a coating on a substrate was proposed in [1]. Such a regime of coating synthesis can be ensured, for instance, with an electron beam unfolded into a line. An analysis of a stationary problem shows that there are different regimes of conversion, which are characterized by different temperatures of the products and different rates of conversion. In this case, there arises a question of conversion stability to small perturbations. It was shown [2, 3] that solid-phase conversion is more unstable to two-dimensional perturbations. In the present paper, we consider a simpler case: one-dimensional perturbations of the form $\exp(\varphi\tau)$. This choice of perturbations is explained by the specific features of the reaction-front behavior described in [4] and caused by a possible dependence of the solid-phase reaction rate on the work of stresses.

Problem for the Case of Small Perturbations. To formulate the problem of stability of the front to small perturbations, we write the nonstationary model with ignored heat release in the heat-conduction equations and without the kinetic equation in the coordinate system fitted to the moving reaction front [1]:

$$
\begin{gathered}
c_\varepsilon\rho\Big(\frac{\partial T_i}{\partial t}+V_n\,\frac{\partial T_i}{\partial x}\Big)=\lambda_T\,\frac{\partial^2 T_i}{\partial x^2}-3K\alpha_T T_i\Big(\frac{\partial\varepsilon_{kk,i}}{\partial t}+V_n\,\frac{\partial\varepsilon_{kk,i}}{\partial x}\Big),
\\
\frac{2\mu}{\lambda+2\mu}\Big(\lambda\,\frac{\partial^2\varepsilon_{yy,i}}{\partial x^2}+2(\lambda+\mu)\,\frac{\partial^2\varepsilon_{xx,i}}{\partial x^2}-3K\alpha_T\,\frac{\partial^2 T_i}{\partial x^2}\Big)+\mu\,\frac{\partial^2\varepsilon_{yy,i}}{\partial x^2}
\\
=\rho\Big(\frac{\partial^2\varepsilon_{xx,i}}{\partial t^2}+2V_n\,\frac{\partial^2\varepsilon_{xx,i}}{\partial t\,\partial x}+V_n^2\,\frac{\partial^2\varepsilon_{xx,i}}{\partial x^2}\Big),
\\
\mu\,\frac{\partial^2\varepsilon_{yy,i}}{\partial x^2}=\rho\Big(\frac{\partial^2\varepsilon_{yy,i}}{\partial t^2}+2V_n\,\frac{\partial^2\varepsilon_{yy,i}}{\partial t\,\partial x}+V_n^2\,\frac{\partial^2\varepsilon_{yy,i}}{\partial x^2}\Big).
\end{gathered}
\tag{1}
$$

Here T is the temperature, x is the spatial coordinate, t is the time, ε_{ij} are the strain-tensor components, c_ε is the specific heat at constant strain, ρ is the density, λ_T is the thermal conductivity, λ,μ are the Lamé coefficients, $K=\lambda+2\mu/3$ is the bulk modulus, α_T is the temperature coefficient of linear expansion, $\varepsilon_{kk}=\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}$, and V_n is the reaction-front velocity.

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In the dimensionless variables

$$\theta_i = \frac{T_i - T_0}{T_{b0} - T_0}, \qquad X = \frac{V_{n0}x}{\kappa_T}, \qquad S_{ij} = \frac{\sigma_{ij}}{\sigma_*}, \qquad e_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_*},$$

where T_{b0} is the temperature of the reaction products in the thermal model, V_{n0} is the velocity of the unperturbed (stationary) wave velocity, σ_{ij} are the stress-tensor components, T_0 is the initial temperature, $\kappa_T = \lambda_T/(c_\varepsilon \rho)$ is the thermal diffusivity, $\sigma_* = 3K\alpha_T(T_{b0} - T_0)$, and $\varepsilon_* = 3K\alpha_T(T_{b0} - T_0)/(\lambda + 2\mu)$, system (1) takes the form

$$\begin{gathered}
\frac{\partial \theta_i}{\partial \tau} + U\,\frac{\partial \theta_i}{\partial X} = \frac{\partial^2 \theta_i}{\partial X^2} - (\theta_b + \sigma)\omega\Big(\frac{\partial e_{kk,i}}{\partial \tau} + U\,\frac{\partial e_{kk,i}}{\partial X}\Big),\\
\gamma_2\Big[\Big(\gamma_1 + \frac{1}{2}\Big)\frac{\partial^2 e_{yy,i}}{\partial X^2} + (\gamma_1 + 1)\,\frac{\partial^2 e_{xx,i}}{\partial X^2} - \frac{\partial^2 \theta_i}{\partial X^2}\Big]\\
= \alpha^2\Big(\frac{\partial^2 e_{xx,i}}{\partial \tau^2} + 2U\,\frac{\partial^2 e_{xx,i}}{\partial \tau\,\partial X} + U^2\,\frac{\partial^2 e_{xx,i}}{\partial X^2}\Big), \qquad (2)\\
\frac{\gamma_2}{2}\,\frac{\partial^2 e_{yy,i}}{\partial X^2} = \alpha^2\Big(\frac{\partial^2 e_{yy,i}}{\partial \tau^2} + 2U\,\frac{\partial^2 e_{yy,i}}{\partial \tau\,\partial X} + U^2\,\frac{\partial^2 e_{yy,i}}{\partial X^2}\Big),\\
i = 1, 2.
\end{gathered}$$

Here, the subscripts $i = 1$ and $i = 2$ refer to the reagents and reaction products,

$$\gamma_1 = \frac{\lambda}{\lambda + 2\mu}, \quad \gamma_2 = \frac{2\mu}{\lambda + 2\mu}, \quad \gamma_1 + \gamma_2 \equiv 1, \quad \omega = \frac{(3K\alpha_T)^2}{\lambda + 2\mu}\,\frac{T_{b0} - T_0}{c_\varepsilon \rho}, \quad \alpha = \frac{V_{n0}^2}{(\lambda + 2\mu)/\rho},$$

$\theta_b = (T_b - T_{b0})/(T_{b0} - T_0)$ is the root of the equation (temperature of the reaction products)

$$\delta_0 \theta_b^2 + (1 + \delta_0 \sigma)\theta_b - 1 = 0; \qquad (3)$$

$$\delta_0 = \frac{\omega(2\gamma_2 - \alpha^2)}{\gamma_2(1 + \gamma_1) - \alpha^2}, \qquad \sigma = \frac{T_0}{T_{b0} - T_0}.$$

The relations between the non-zero components of the stress and strain tensors

$$\begin{gathered}
S_{yy} = \gamma_2[\gamma_1 e_{xx} + (1 + \gamma_1)e_{yy} - \theta],\\
S_{xx} = \gamma_2[\gamma_1 e_{yy} + (1 + \gamma_1)e_{xx} - \theta], \qquad (4)\\
e_{kk} = \theta_i + \gamma_2[e_{yy,i} + e_{xx,i}]
\end{gathered}$$

are valid in the nonstationary wave for both the reagents and the reaction products.

The conditions on the perturbed boundary $X = \zeta$ can be presented in the dimensionless variables as

$$\begin{gathered}
\theta_1 = \theta_2 = \theta_s, \qquad \frac{d\theta_1}{dX} = \frac{d\theta_2}{dX} + U \equiv \frac{d\theta_2}{dX} + \frac{V_n}{V_{n0}},\\
e_{xx,1} = e_{xx,2}, \qquad \frac{de_{xx,1}}{dX} = \frac{de_{xx,2}}{dX} + \frac{\gamma_2 U}{\gamma_1(1 + \gamma_1) - \alpha^2},\\
e_{yy,1} = e_{yy,2}, \qquad \frac{de_{yy,1}}{dX} = \frac{de_{yy,2}}{dX}, \qquad (5)\\
\frac{d\zeta}{d\tau} = 1 - U.
\end{gathered}$$

The last equality is the condition of the complete consumption of the substance in the reaction zone [2, 3].

The solution of the nonstationary problem (2), (5) is sought in the form

$$U = U^0 + U', \qquad \theta_i = \theta_i^0 + \theta_i', \quad \ldots\ , \qquad (6)$$

where the prime corresponds to small perturbations and $U^0 \equiv 1$.

Substituting Eq. (6) into Eqs. (2), taking into account the solution of the stationary problem [1], and neglecting small terms, we can show that a system of equations similar to Eqs. (2), but with $U = 1$ is valid for the perturbations. Small terms also include terms of the form $U'\, d\theta_i^0/dX$, because we have $d\theta_1^0/dX \ll 1$ and $d\theta_2^0/dX = 0$ far from the reaction zone.

To formulate the conditions for perturbations on the boundary $X = \zeta$, we present the perturbed velocity of the front as a series in the vicinity of the unperturbed value of V_{n0}. In physical variables, we have

$$V_n = V_{n0} + \Big(\frac{\partial V_n}{\partial T}\Big)_0 (T - T^0) + \Big(\frac{\partial V_n}{\partial \Pi}\Big)_0 (\Pi - \Pi^0),$$

where $\Pi = -\sum\limits_{(i,j)} \sigma_{ij}\varepsilon_{ij}$; Π^0 and T^0 are the work of stresses and the temperature in the stationary wave, respectively. Passing to dimensionless variables, we write

$$U = V_n/V_{n0} = 1 + z_1(\theta_s - \theta_s^0) - z_2(\bar{\Pi}_s - \bar{\Pi}_s^0), \tag{7}$$

where $\bar{\Pi} = \Pi/(\sigma_*\varepsilon_*)$; the subscript s indicates the quantities on the surface; z_1 and z_2 are the coefficients of sensitivity of the reaction rate to temperature [2, 3] and to the work of stresses [5]:

$$z_1 = \frac{T_{b0} - T_0}{V_{n0}} \Big(\frac{\partial V_n}{\partial T}\Big)_0, \qquad z_2 = -\frac{\sigma_*\varepsilon_*}{V_{n0}} \Big(\frac{\partial V_n}{\partial \Pi}\Big)_0.$$

Therefore, we have

$$\bar{\Pi}' = \bar{\Pi}_s - \bar{\Pi}_s^0 = -\gamma_2[2\gamma_1 e_{xx}^0 e'_{yy} + 2(1+\gamma_1) e_{xx}^0 e'_{xx} - \theta'_s e_{xx}^0 - \theta_s^0 (e'_{xx} + e'_{yy})].$$

Using Eq. (4) and the stationary solution [1], we obtain

$$\bar{\Pi} = -\gamma_2 \theta_b \Big(\frac{\gamma_2(1+\gamma_1) + \alpha^2}{\gamma_2(1+\gamma_1) - \alpha^2}\, e'_{xx,s} - \frac{\gamma_2 \theta'_s}{\gamma_2(1+\gamma_1) - \alpha^2}\Big).$$

Expanding all quantities in the boundary conditions (5) into series in the vicinity of the unperturbed boundary and taking into account the conditions of the stationary problem, we obtain the conditions for perturbations [2, 3]:

$$\begin{gathered}
\frac{d\theta_1^0}{dX}\,\zeta + \theta'_1 = \frac{d\theta_2^0}{dX}\,\zeta + \theta'_2 = \theta'_s, \\
\frac{d^2\theta_1^0}{dX^2}\,\zeta + \frac{\partial\theta'_1}{\partial X} = \frac{d^2\theta_2^0}{dX^2}\,\zeta + \frac{\partial\theta'_2}{\partial X} + z_1\theta'_s - z_2\bar{\Pi}'_s, \\
\frac{de_{xx,1}^0}{dX}\,\zeta + e'_{xx,1} = \frac{de_{xx,2}^0}{dX}\,\zeta + e'_{xx,2} = e'_{xx,s}, \\
\frac{d^2 e_{xx,1}^0}{dX^2}\,\zeta + \frac{\partial e'_{xx,1}}{\partial X} = \frac{d^2 e_{xx,2}^0}{dX^2}\,\zeta + \frac{\partial e'_{xx,2}}{\partial X} + \gamma_2\,\frac{z_1\theta'_s - z_2\bar{\Pi}'_s}{\gamma_2(1+\gamma_1) - \alpha^2}, \\
\frac{d\zeta}{d\tau} = -(z_1\theta'_s - z_2\Pi'_s), \\
\frac{de_{yy,1}^0}{dX}\,\zeta + e'_{yy,1} = \frac{de_{yy,2}^0}{dX}\,\zeta + e'_{yy,2}, \qquad \frac{d^2 e_{yy,1}^0}{dX^2}\,\zeta + \frac{\partial e'_{yy,1}}{\partial X} = \frac{d^2 e_{yy,2}^0}{dX^2}\,\zeta + \frac{\partial e'_{yy,2}}{\partial X}.
\end{gathered} \tag{8}$$

Thus, the problem for perturbations is formulated.

Conditions of Stability. The stability conditions for the stationary front are obtained by analyzing the evolution of the perturbation defined in the form

$$\zeta = a \exp(\varphi\tau), \tag{9}$$

where a is the perturbation amplitude and φ is the complex frequency.

The solution of the problem for perturbations is sought in the form

$$\theta_i' = f_i \exp(\varphi\tau \pm r_i X),$$
$$e'_{xx,i} = h_i \exp(\varphi\tau \pm r_i X), \qquad e'_{yy,i} = g_i \exp(\varphi\tau \pm r_i X), \tag{10}$$

where the logarithmic decrement r_1 and the plus sign correspond to the domain of the reagents ($X < 0$), while the logarithmic decrement r_2 and the minus sign refer to the domain of the reaction products ($X > 0$).

Substituting Eq. (9) into the system of equations for perturbations (5), where $U = 1$, we obtain a system of algebraic equations with respect to the amplitudes f_1, h_1, g_1 and f_2, h_2, g_2:

$$f_i[(\varphi \pm r_i)(1 + \omega(\theta_b + \sigma)) - r_i^2] + (h_i + g_i)\omega(\theta_b + \sigma)\gamma_2(\varphi \pm r_i) = 0,$$
$$-f_i\gamma_2 r_i^2 + h_i[\gamma_2(1 + \gamma_1)r_i^2 - \alpha^2(\varphi \pm r_i)^2] + g_i\gamma_2(\gamma_1 + 1/2)r_i^2 = 0; \tag{11}$$

$$g_i[\gamma_2 r_i^2/2 - \alpha^2(\varphi \pm r_i)^2] = 0. \tag{12}$$

It follows from Eqs. (12) that $g_1 = 0$ and $g_2 = 0$.

The condition of solvability of Eqs. (11) is the zero values of the determinants composed from the coefficients for these equations:

$$[(\varphi \pm r_i)(1 + \omega(\theta_b + \sigma)) - r_i^2][\gamma_2(1 + \gamma_1)r_i^2 - \alpha^2(\varphi \pm r_i)^2] + \omega(\theta_b + \sigma)\gamma_2^2 r_i^2(\varphi \pm r_i) = 0, \tag{13}$$

$$i = 1, 2.$$

This equation can be resolved analytically for two particular cases with $\omega = 0$ or $\alpha = 0$, i.e., for the uncoupled problem, and for the case where the inertial forces are negligibly small.

As $g_1 = 0$ and $g_2 = 0$, there are five conditions left on the interface. Using Eqs. (9) and (10) and taking into account the solution of the stationary problem [1], we find a system of linear equations with respect to the perturbation amplitudes f_i, h_i, and g_i from the first five linearized boundary conditions (8). The condition of solvability of this system is the equation

$$\begin{vmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 + \delta & r_1 & r_2 - \beta & 0 & -\kappa \\ \gamma & 0 & 0 & 1 & -1 \\ \gamma(1 + \delta) & 0 & -\beta\gamma & r_1 & r_2 - \kappa\gamma \\ \varphi & 0 & \beta & 0 & \kappa \end{vmatrix} = 0,$$

where

$$\gamma = \frac{\gamma_2}{\gamma_2(1 + \gamma_1) - \alpha^2}, \qquad \beta = z_1 - \frac{z_2\gamma_2^2\theta_b}{\gamma_2(1 + \gamma_1) - \alpha^2},$$

$$\kappa = z_2\gamma_2\theta_b\frac{\gamma_2(1 + \gamma_1) + \alpha^2}{\gamma_2(1 + \gamma_1) - \alpha^2}, \qquad \delta = \delta_0(\theta_b + \sigma).$$

The latter relation is written with allowance for the equality $(1 + \delta)\theta_b \equiv 1$ following from Eq. (3).

Expanding the determinant, we obtain

$$(r_1 + r_2)[\varphi(r_1 + r_2) + (\beta + \kappa\gamma)(r_1 - \varphi - 1 - \delta)] = 0. \tag{14}$$

By analyzing Eqs. (13) and (14), we can find the conditions of stability of the stationary front to one-dimensional perturbations.

Investigation of Stability for Particular Values of Problem Parameters. Let us consider the following cases.

1. At $\alpha = 0$, $z_2 = 0$, and $\omega = 0$, we obtain the known result. Indeed, in this case, it follows from Eq. (13) that $\varphi + r_1 - r_1^2 = 0$ and $\varphi - r_2 - r_2^2 = 0$. Then, Eq. (14) yields

$$\varphi[4\varphi^2 + (-z_1^2 + 4z_1 + 1)\varphi + z_1] = 0.$$

Therefore, we have

$$r_{1,2} = (\sqrt{1+4\varphi} \pm 1)/2,$$

and the condition of stability of the stationary regime takes the form [2, 3]

$$0 < z_1 < z_{1*} = 2 + \sqrt{5}.$$

In this case, we have

$$\theta_b = 1, \qquad \theta_1^0 = \exp(X), \qquad \theta_2^0 = 1,$$

which also follows from the classical solution. In this case, we have

$$e^0_{xx,i} = \frac{\theta_i^0}{1+\gamma_1}, \qquad e^0_{zz,i} = \frac{\theta_i^0}{1+\gamma_1}, \qquad S^0_{xx,i} \equiv 0, \qquad S^0_{yy,i} = -\frac{\gamma_2\theta_i^0}{1+\gamma_1}.$$

If $\alpha \neq 0$, then Eqs. (13) have additional roots

$$r_{1,2} = \varphi\alpha/(\sqrt{\gamma_2(1+\gamma_1)} \pm \alpha)$$

at $\alpha < \sqrt{\gamma_2(1+\gamma_1)} \equiv \gamma_*$. The solutions obtained correspond to the wave of mechanical perturbations, which accompanies the solid-phase combustion front.

The condition of the loss of stability of the mechanical wave is obtained by analyzing the equation

$$\varphi^2 - \frac{\gamma_* - \alpha}{2\alpha}\, z_1\varphi - \frac{\gamma_*^2 - \alpha^2}{2\gamma_*\alpha} = 0. \tag{15}$$

It follows from Eq. (15) that

$$\varphi_{1,2} = \frac{1}{2}\,\frac{\gamma_* - \alpha}{2\alpha}\, z_1\Big(1 \pm \sqrt{1 + 2\,\frac{\alpha}{\gamma_*}\,\frac{\gamma_* + \alpha}{\gamma_* - \alpha}\,\frac{1}{z_1^2}}\,\Big).$$

If $\alpha < \gamma_* = \sqrt{\gamma_2(1+\gamma_1)} = \sqrt{\gamma_2(2-\gamma_2)}$, then Eq. (15) has two real roots; one of them is positive ($\varphi_1 > 0$), and the other is negative ($\varphi_2 < 0$). This allows us to conclude that the wave of mechanical perturbations generated by the slow combustion wave is absolutely unstable.

If $\alpha > \gamma_*$, then $\mathrm{Re}\,(\varphi) < 0$, and the wave of mechanical perturbations should be absolutely stable. However, there is a condition of existence of the combustion wave in this range of parameters $\alpha < \gamma_*$ [1]. Therefore, such a wave of mechanical perturbations does not exist. In other words, the domain of existence of the stationary combustion wave does not coincide with the domain of stability of the wave of mechanical perturbations (and even has no intersections with the latter).

2. At $\alpha = 0$, $z_2 = 0$, and $\omega \neq 0$ (i.e., for a definitely slow combustion wave), we have

$$\delta = \frac{2\omega}{1+\gamma_1}\,(\theta_b + \sigma);$$

therefore, the only value of temperature is

$$\theta_b = \Big[\sqrt{(1+B\sigma)^2 + 4B} - (1+B\sigma)\Big]/(2B).$$

Here, $B = 2\omega/(1+\gamma_1) \equiv 2\omega/(2-\gamma_2)$. The dependence of θ_b on ω is shown in Fig. 1a.

The dispersion relations take the form

$$[(\varphi + r_1)(1+A) - r_1^2](1+\gamma_1) + A(\varphi + r_1) = 0,$$

$$[(\varphi - r_2)(1+A) - r_2^2](1+\gamma_1) + A(\varphi - r_2) = 0,$$

where $A = \omega(\theta_b + \sigma) > 0$. The positive roots of these equations are

$$r_{1,2} = \frac{1}{2(1+\gamma_1)}\Big[\sqrt{C^2 + 4(1+\gamma_1)C\varphi} \pm C\Big],$$

where $C = (1+A)(1+\gamma_1) + A(1-\gamma_1) = 1 + 2A + \gamma_1 > 0$.

The condition of stability of the stationary front can be obtained by analyzing the equation

$$a_0\varphi_3 + a_1\varphi^2 + a_2\varphi + a_3 = 0,$$

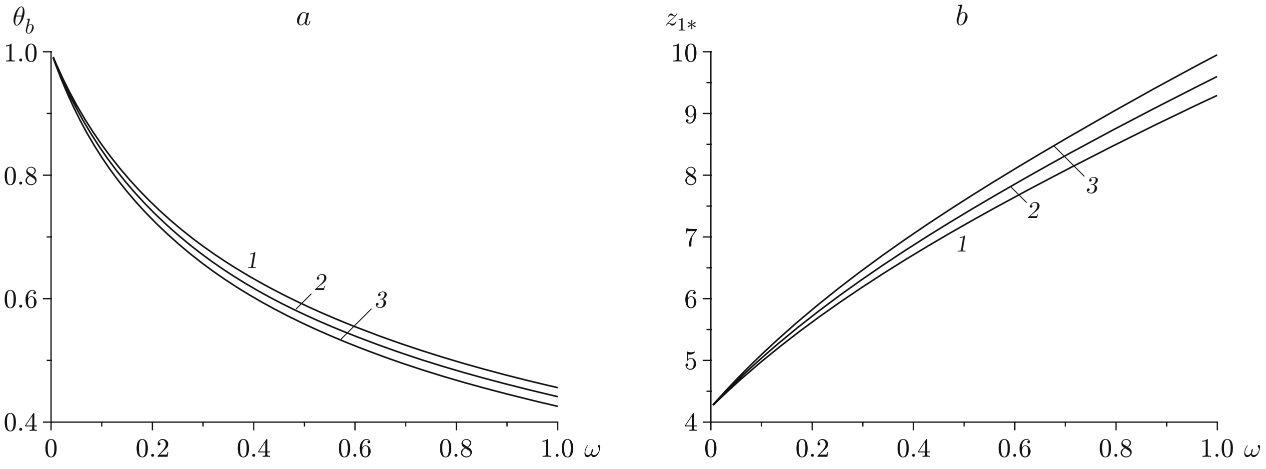

Fig. 1. Temperature of the products (a) and stable combustion limit (b) versus the coupling coefficient ($\alpha = 0$, $z_2 = 0$, and $\sigma = 0.35$): $\gamma_2 = 0.65$ (1), 0.75 (2), and 0.85 (3).

where

$$a_0 = 16(1+\gamma_1)C, \qquad a_1 = 16z_1(1+\gamma_1)C + 4C^2 - 4z_1^2(1+\gamma_1)^2,$$

$$a_2 = 4z_1^2[(1+\gamma_1)C - 2(1+\gamma_1)^2 - (2A-1-\gamma_1)(1+\gamma_1)] + 4z_1C^2,$$

$$a_3 = z_1^2[C^2 - 4(1+\gamma_1)^2 - 4(2A-1-\gamma_1)(1+\gamma_1) - (2A-1-\gamma_1)^2].$$

The condition of the loss of stability is based on the change in the sign of the coefficient a_1, leading to the emergence of roots with a positive real part under the condition

$$z_1 > z_{1*} = \frac{C}{1+\gamma_1}\,(2+\sqrt{5}\,) \equiv \Big(1 + \frac{2\omega(\theta_b+\sigma)}{1+\gamma_1}\Big)(2+\sqrt{5}\,) > z_{1*}^0.$$

Thus, the greater the value of the coupling coefficient, the lower the temperature of the products, but the broader the domain of stable combustion (see Fig. 1).

3. At $z_2 \neq 0$, but $\omega = 0$ and $\alpha = 0$, we obtain

$$\beta + \kappa\gamma \equiv z_1,$$

and the condition of stability of the stationary combustion wave remains unchanged.

If $\alpha \neq 0$, then (as in [4–6]) we obtain two limits of stable combustion (upper and lower limits) even if $\omega = 0$. In this case, we have

$$\theta_b = 1, \qquad \beta + \kappa\gamma = z_1 + z_2\,2\gamma_2^2\alpha^2/(\gamma_*^2 - \alpha^2)^2.$$

If $z_2 > 0$, i.e., the stresses induce an increase in the reaction rate [see Eq. (7)], then the dependence of the reaction rate on the work of stresses leads to a decrease in the stable combustion limit:

$$z_1 < 2 + \sqrt{5} - \Omega z_2.$$

Here, $\Omega = 2\gamma_2^2\alpha^2/(\gamma_*^2 - \alpha^2)^2$. Obviously, this inequality is meaningful if $z_2 < (2+\sqrt{5}\,)/\Omega$.

If $z_2 < 0$, i.e., the stresses lead to reaction deceleration, then we have

$$z_1^b < z_1 < z_1^a.$$

Here, $z_1^a = 2 + \sqrt{5} + \Omega|z_2|$ and $z_1^b = 2 - \sqrt{5} + \Omega|z_2|$. Obviously, for the lower limit of stable combustion to exist, the following inequality should be satisfied:

$$z_1^b > (\sqrt{5} - 2)/\Omega.$$

The domain of existence of stable stationary regimes of solid-phase conversion under the conditions of a plane stress state is shown in Fig. 2. Stable propagation of the combustion front occurs in the domains between the

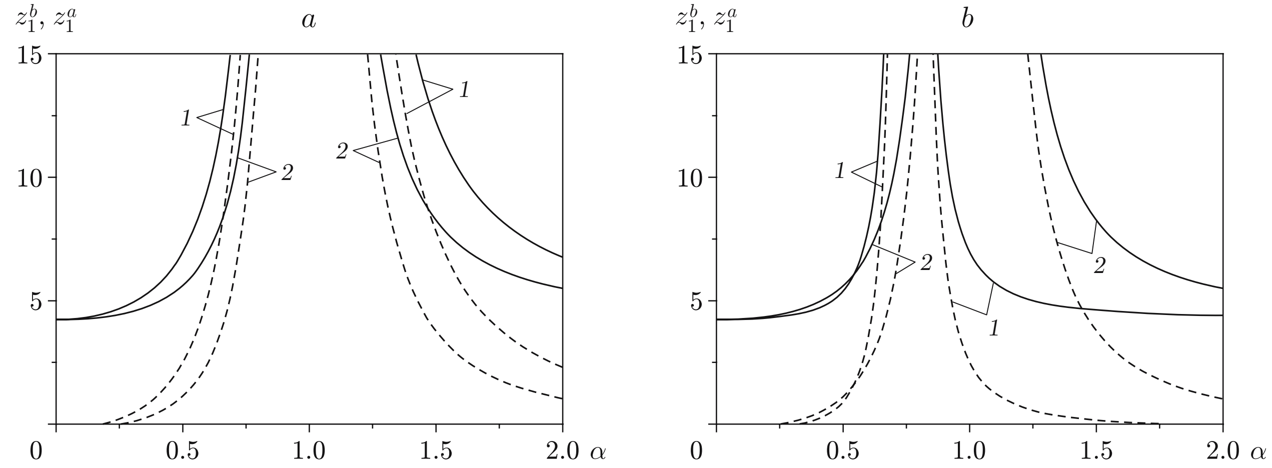

Fig. 2. Limits of stable propagation of the combustion front for different model parameters ($\omega = 0$ and $z_2 \neq 0$): (a) $\gamma_2 = 0.85$ and $z_2 = -4$ (1) and 2 (2); (b) $z_2 = -2$ and $\gamma_2 = 0.35$ (1) and 0.85 (2); the solid and dashed curves show the values of z_1^a and z_1^b, respectively.

solid and dashed curves with identical numbers. It follows from Fig. 2 that there are domains of stable propagation of both slow solid-phase combustion regimes ($\alpha < 1$) and high-velocity regimes. The greater the dependence of the reaction rate on stresses, the higher the domain of existence of stable regimes (curves 1 and 2 in Fig. 2a). A decrease in γ_2 shifts the beginning of the domain of stable regimes toward lower velocities (Fig. 2b).

Conclusions. Thus, using the model proposed in [1], we demonstrated that the conditions of sample fixation in the experiment exert a significant effect on stability of the stationary conversion front. In any case, however, stability can be characterized by two limits (upper and lower), which have different physical meanings. The substrate can affect both the regimes of reaction-front propagation and their stability, not only through heat removal from the reaction zone, which is taken into account in the model considered. Taking into account the properties of the coating and substrate leads to the emergence of new parameters; the basic new parameters are the ratio of the specific heats of materials and the ratio of their temperature coefficients of linear expansion. Finally, this leads to re-defining of the effective parameters in the constitutive equations and the domains of existence of various regimes of conversion. For instance, the parameter $c_{\varepsilon,c}[1 + hc_\varepsilon/(h_c c_{\varepsilon,c})]$ appears in the model instead of c_ε, and the parameter $\alpha_{T,c}[1 + h\alpha_T/(h_c\alpha_{T,c})]$ appears instead of α_T. In this case, an additional study of the character of the stationary solutions and the conditions of the loss of their stability is needed.

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